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A Simple Adaptive Control of Electrically Driven Flexible-Joint Robots Using Function Approximation Techniques

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***Abstract*— In this paper, we propose a simple adaptive control approach for uncertain flexible-joint robots including motor dynamics. The dynamic surface method is applied to design the simple controller for electrically driven flexible-joint (EDFJ) robots, and the uncertainties in the robot and motor dynamics are compensated by using the adaptive function approximation technique. We prove that all signals in the controlled closed-loop system are uniformly ultimately bounded. Simulation results for three-link EDFJ manipulators are provided to validate the effectiveness of the proposed control system.**

I. Introduction

During the past several years, the tracking control of the flexible joint (FJ) robots has attracted many researchers due to the joint flexibility. There are many works using various control techniques such as PD control [1], sliding mode control [2], [3], fuzzy control [4], [5], neural network (NN) control [6], [7], and backstepping control [9]- [11]. However, all of these schemes have ignored the dynamics coming from electric motors which should be required to implement the FJ robots in the real environment. Considering motor dynamics makes difficult and complex to design the controller for electrically driven flexible-joint (EDFJ) robots which is the fifth-order nonlinear system. Even if a recent result were reported for EDFJ robots, the uncertainties of the joint flexibility were not considered [12].

In this paper, we propose a simple adaptive control approach for EDFJ robots with uncertainties and disturbances. The dynamic surface method [13] which can solve the "explosion of complexity" problem of the backstepping technique is applied to design a simple controller of EDFJ robots. In addition, the function approximation technique using self-recurrent wavelet neural networs (SRWNNs) [14] and the adaptive technique are employed to compensate the model uncertainties and disturbances. From Lyapunov stability analysis, it is shown that all signals in a closed-loop adaptive system are uniformly ultimately bounded. Finally, we simulate a uncertain three-link EDFJ manipulator with complex nonlinear functions to demonstrate the simplicity and the robustness of the proposed control scheme.

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This paper is organized as follows. In Section 2, we introduce the model and basic properties of EDFJ robot systems with uncertainties. In Section 3, the function approximation technique using SRWNN is presented and a simple adaptive control system for solving the robust control problem of the EDFJ robot system is proposed. In addition, the stability, robustness, and performance of the proposed control system are analyzed based on Lyapunov stability theorem. Simulation results are discussed in Section 4. Finally, Section 5 gives some conclusions.

II. Problem Formulation

The dynamic model of an uncertain n-link EDFJ robot consists of robot dynamics, joint flexibility, and motor dynamics described by using the following forms:

$$\begin{aligned} &M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F\dot{q} \\ &\qquad + K_m(q-q_m) + \Upsilon_r(q,\dot{q},q_m) = 0 \qquad (1) \\ &J\ddot{q}_m + B\dot{q}_m + K_m(q_m - q) \\ &\qquad + \Upsilon_a(q_m,\dot{q}_m,q,i_e) = Hi_e \qquad (2) \\ &\bar{L}\dot{i}_e + Ri_e + K_e\dot{q}_m + \Upsilon_e(\dot{q}_m,i_e) = u \qquad (3) \end{aligned}$$

where

$$\begin{aligned} \Upsilon_r(q,\dot{q},q_m) &= -M(q)\bar{M}^{-1}(q)\{\bar{K}_m(q_m-q) \\ &\quad - T_r - \bar{F}\dot{q} - \bar{G}(q) - \bar{C}(q,\dot{q})\dot{q}\} \\ &\quad + \{K_m(q_m-q) - F\dot{q} - G(q) - C(q,\dot{q})\dot{q}\}, \\ \Upsilon_a(q_m,\dot{q}_m,q,i_e) &= -J\bar{J}^{-1}\{Hi_e - T_a \\ &\quad - \bar{K}_m(q_m-q) - \bar{B}\dot{q}_m\} \\ &\quad + \{Hi_e - K_m(q_m-q) - B\dot{q}_m\}, \end{aligned}$$

and

$$\Upsilon_e(\dot{q}_m,i_e) = (\bar{R}-R)i_e + (\bar{K}_e - K_e)\dot{q}_m + T_e$$

including the external disturbances T_r T_a, and $T_e \in \mathcal{R}^n$, denote the uncertainty terms of robot dynamics, joint flexibility, and actuator dynamics of the EDFJ robot system, respectively. Here, q, $\dot{q}$, $\ddot{q} \in \mathcal{R}^n$ denote the link position, velocity, and acceleration vectors, respectively. $M(q) \in \mathcal{R}^{n\times n}$ is the nominal inertia matrix, $C(q,\dot{q}) \in \mathcal{R}^{n\times n}$ denotes the nominal Coriolis-centripetal matrix, $G(q) \in \mathcal{R}^n$ is the nominal gravity vector, and $F \in \mathcal{R}^{n\times n}$ is a nominal diagonal, positive definite matrix representing the coefficient of friction at each joint. q_m, $\dot{q}_m$, $\ddot{q}_m \in \mathcal{R}^n$ denote the actuator position, velocity, and acceleration vectors, respectively. The nominal constant positive definite, diagonal matrices $K_m \in \mathcal{R}^{n\times n}$, $J \in \mathcal{R}^{n\times n}$, and $B \in \mathcal{R}^{n\times n}$ represent the joint flexibility, the actuator inertia, and the natural damping term, respectively. $H \in \mathcal{R}^{n\times n}$ is a nominal invertible diagonal matrix which characterizes the electromechanical conversion between current and torque. $i_e \in \mathcal{R}^n$ is the armature current vector of n dc joint motors, $\bar{L}$ are an actual positive definite constant diagonal matrix denoting the electrical inductance of the motors. R and $K_e \in \mathcal{R}^{n\times n}$ are nominal positive definite constant diagonal matrices denoting electrical resistance and back electromotive force constant of the motors, respectively. The control vector $u \in \mathcal{R}^n$ is used as the torque input at each actuator.

Assumption 1: Suppose that the nominal matrices $M(q)$, $C(q,\dot{q})$, $G(q)$, F, K_m, J, B, H, R, and K_e are only known, but the actual matrices $\bar{M}(q)$, $\bar{C}(q,\dot{q})$, $\bar{G}(q)$, $\bar{F}$, $\bar{K}_m$, $\bar{J}$, $\bar{B}$, $\bar{H}$, $\bar{L}$, $\bar{R}$, and $\bar{K}_e$ with model uncertainties, and the external disturbances T_r T_a, and T_e are unknown.

Assumption 2: The system states q, $\dot{q}$, q_m, $\dot{q}_m$, and i_e are all available for feedback.

Assumption 3: The desired trajectory vector q_d, its first and second derivatives $\dot{q}_d$, $\ddot{q}_d$ are only available, and bounded.

Property 1: *[9]* The link inertia matrix $M(q)$ is symmetric, positive definite, and both $M(q)$ and $M^{-1}(q)$ are uniformly bounded.

Property 2: $\|M^{-1}(q)K_m\|_2 \le M_m$ where M_m is a known positive constant, and $\|\cdot\|_2$ denotes the matrix induced two-norm.

Property 2 is reasonable due to *Property* 1 and a constant positive definite matrix K_m.

Define the state space variables as $x_1 = q$, $x_2 = \dot{q}$, $x_3 = q_m$, $x_4 = \dot{q}_m$, and $x_5 = i_e$. Then, the uncertain EDFJ robot system is described as the following state-space forms:

$$\dot{x}_1 = x_2, \tag{4}$$

$$\dot{x}_2 = M^{-1}(x_1)[-C(x_1,x_2) - G(x_1) - F(x_2) - K_m x_1 + K_m x_3] + \Xi_r(x_r), \tag{5}$$

$$\dot{x}_3 = x_4, \tag{6}$$

$$\dot{x}_4 = J^{-1}[-Bx_4 - K_m(x_3 - x_1) + Hx_5] + \Xi_a(x_a), \tag{7}$$

$$\bar{L}\dot{x}_5 = -Rx_5 - K_e x_4 + \Xi_e(x_e) + u \tag{8}$$

where $x_r = [x_1^T\ x_2^T\ x_3^T]^T$, $x_a = [x_1^T\ x_3^T\ x_4^T\ x_5^T]^T$, $x_e = [x_4^T\ x_5^T]^T$, $\Xi_r = -M^{-1}(x_1)\Upsilon_r$, $\Xi_a = -J^{-1}\Upsilon_a$, and $\Xi_e = -\Upsilon_e$.

The objective of this paper is to design a simple adaptive control law u for the state vector x_1 of EDFJ robots to track the desired trajectory vector q_d under *Assumptions* 1-3.

III. MAIN RESULTS

A. Function Approximation Using SRWNNs

To compensate the unknown uncertainty terms, we use the self-recurrent wavelet neural network (SRWNN) and the adaptive technique. That is, the uncertainty terms $\Xi_j(x_j)$ $(j = r,a,e)$ are approximated by SRWNN and the unknown actual diagonal constant matrix $\bar{L}$ is estimated by the adaptive technique.

The SRWNN consists of the four layer, i.e, an input layer, a mother wavelet layer including self-loop weights, a product layer, and an output layer. See [14] for the detail structure of the SRWNN.

According to the powerful approximation ability, the SRWNN systems $\widehat{\Xi}_j(\cdot)$ can approximate the uncertainty terms $\Xi_j(\cdot)$ to a sufficient degree of accuracy over compact sets $\mathcal{K}_{x_j}$ as follows:

$$\begin{aligned}\Xi_j(x_j) &= \widehat{\Xi}_j(x_j|W_j^*) + \varepsilon_j(x_j)\\ &= \widehat{\Xi}_j(x_j|\widehat{W}_j) + [\widehat{\Xi}_j(x_j|W_j^*) - \widehat{\Xi}_j(x_j|\widehat{W}_j)]\\ &\quad + \varepsilon_j(x_j)\end{aligned} \tag{9}$$

where $j = r,a,e$, $x_j \in \mathcal{K}_{x_j}$ are the inputs of SRWNN systems, $\varepsilon_j(x_j)$ denote reconstruction errors, $\widehat{W}_j = \mathrm{diag}[\widehat{W}_{j,i}]$ $(i = 1,2,\cdots,n)$ are estimated weighting matrices, and W_j^* are optimal weighting matrices. Here, diag$[\cdot]$ denotes a diagonal matrix, and $\widehat{W}_{j,i}$ are estimated weight vectors. The optimal weighting matrices W_j^* for SRWNNs $\widehat{\Xi}_j(\cdot)$ are defined as $W_j^* = \arg\min_{\widehat{W}_j}[\sup_{x_j\in\mathcal{K}_{x_j}} \|\Xi_j(x_j) - \widehat{\Xi}_j(x_j|\widehat{W}_j)\|]$.

Assumption 4: [15] Assume that the optimal weight matrices are bounded as $\|W_j^*\|_F \le W_{j,M}$, where $j = r,a,e$, $\|\cdot\|_F$ denotes the Frobenius norm.

Note that the bounded values $W_{j,M}$ are not required to implement the controller proposed in this paper. These values are used only for the stability analysis of the proposed control system. Taking the Taylor series expansion of $\widehat{\Xi}_j(x_j|W_j^*)$ around $\widehat{W}_j$ for the training of all weights of the SRWNNs, respectively, we can obtain [16]

$$\widehat{\Xi}_j(x_j|W_j^*) - \widehat{\Xi}_j(x_j|\widehat{W}_j) = \widetilde{W}_j^T\Theta_j + H_j(W_j^*,\widehat{W}_j) \tag{10}$$

where $j = r,a,e$, $\widetilde{W}_j(t) = W_j^* - \widehat{W}_j(t)$, $\Theta_j = \left[\frac{\partial\widehat{\Xi}_{j,1}}{\partial\widehat{W}_{j,1}}\ \ \frac{\partial\widehat{\Xi}_{j,2}}{\partial\widehat{W}_{j,2}}\ \ \cdots\ \ \frac{\partial\widehat{\Xi}_{j,n}}{\partial\widehat{W}_{j,n}}\right]^T$, $H_j(W_j^*,\widehat{W}_j)$ are high-order terms. Substituting (10) into (9), we obtain

$$\Xi_j(x_j) = \widehat{\Xi}_j(x_j|\widehat{W}_j) + \widetilde{W}_j^T\Theta_j + \alpha_j \tag{11}$$

$$\|\alpha_j\| \le \rho_j \tag{12}$$

where $j = r,a,e$, $\alpha_j = H_j(W_j^*,\widehat{W}_j) + \varepsilon_j(x_j)$. $\rho_j > 0$ are unknown values used only for the stability analysis of the proposed control system.

B. Adaptive Controller Design

In this section, we present the dynamic surface design approach for designing a simple adaptive control of EDFJ robots. The proposed control system is designed step by step.

Step 1) Consider the link dynamics (4)-(5). Define the first error surface vector s_1 as

$$s_1 = x_2 - \dot{q}_d + \Lambda(x_1 - q_d) \tag{13}$$

where Λ denotes a positive definite diagonal matrix. Differentiating (13) yields

$$\begin{aligned}\dot{s}_1 = M^{-1}(x_1)[&-C(x_1,x_2) - G(x_1) - F(x_2)\\ &- K_m x_1 + K_m x_3] + \Xi_r(x_r) - \ddot{q}_d\\ &+ \Lambda(x_2 - \dot{q}_d).\end{aligned}$$

Then, we choose the virtual control law ν_1 as

$$\begin{aligned}\nu_1 = K_m^{-1}[&K_m x_1 + C(x_1,x_2) + G(x_1) + F(x_2) + M\\ &\times\{-k_1 s_1 - \widehat{\Xi}_r(x_r|\widehat{W}_r) + \ddot{q}_d - \Lambda(x_2 - \dot{q}_d)\}]\end{aligned} \tag{14}$$

where $k_1 > 0$ is a constant. $\widehat{W}_r$ is the estimate of the weighting matrix W_r, and is updated by the adaptation law using a σ-modification [17] as follows:

$$\dot{\widehat{W}}_{r,i} = \lambda_{1,i}\Theta_{r,i}s_{1,i} - \sigma_1\lambda_{1,i}\widehat{W}_{r,i} \tag{15}$$

where $i = 1,\ldots,n$, λ_1 is a tuning gain matrix, and $\sigma_1 > 0$. Here, $\widehat{W}_{r,i}$ and $\lambda_{1,i}$ the ith diagonal element of $\widehat{W}_r$ and λ_1, respectively. $\Theta_{r,i}$ and $s_{1,i}$ are the ith element of Θ_r and s_1, respectively.

For the filtered virtual controller ν_{1f}, the virtual controller ν_1 is passed through the first-order low pass filter

$$\tau_1\dot{\nu}_{1f} + \nu_{1f} = \nu_1, \quad \nu_{1f}(0) = \nu_1(0) \tag{16}$$

with a time constant τ_1.

Step 2) Consider the equation (6). The surface error is defined as $s_2 = x_3 - \nu_{1f}$, and its derivative is $\dot{s}_2 = x_4 - \dot{\nu}_{1f}$. Then, choose the virtual control law using (16) as

$$\nu_2 = -k_2 s_2 + (\nu_1 - \nu_{1f})/\tau_1 \tag{17}$$

where $k_2 > 0$ is a constant. To obtain the filtered virtual controller ν_{2f}, we pass ν_2 through a first-order low pass filter with a time constant $\tau_2 > 0$ as follows:

$$\tau_2\dot{\nu}_{2f} + \nu_{2f} = \nu_2, \quad \nu_{2f}(0) = \nu_2(0). \tag{18}$$

Step 3) Consider the equation (7). Define the surface error, with the filtered virtual control vector ν_{2f}, as $s_3 = x_4 - \nu_{2f}$. Then, differentiating it and substituting (7) yields

$$\begin{aligned}\dot{s}_3 &= \dot{x}_4 - \dot{\nu}_{2f}\\ &= J^{-1}[-Bx_4 - K_m(x_3 - x_1) + Hx_5]\\ &\quad + \Xi_a(x_a) - \dot{\nu}_{2f},\end{aligned} \tag{19}$$

Then, we choose the virtual control law ν_3 as

$$\begin{aligned}\nu_3 = H^{-1}[&K_m(x_3 - x_1) + Bx_4 + J\{-k_3 s_3\\ &- \widehat{\Xi}_a(x_a|\widehat{W}_a) + (\nu_2 - \nu_{2f})/\tau_2\}]\end{aligned} \tag{20}$$

where k_3 is a positive constant. $\widehat{W}_a$ is the estimate of the weighting matrix W_a, and is updated by the adaptation law

$$\dot{\widehat{W}}_{a,i} = \lambda_{2,i}\Theta_{a,i}s_{3,i} - \sigma_2\lambda_{2,i}\widehat{W}_{a,i} \tag{21}$$

where $i = 1,\ldots,n$, λ_2 is a tuning gain matrix, and $\sigma_2 > 0$. Here, $\widehat{W}_{a,i}$ and $\lambda_{2,i}$ are the ith diagonal element of $\widehat{W}_a$ and λ_2, respectively. $\Theta_{a,i}$ and $s_{3,i}$ are the ith element of Θ_a and s_3, respectively. In addition, the filtered virtual control law ν_{3f} is obtained by the following first-order filter:

$$\tau_3\dot{\nu}_{3f} + \nu_{3f} = \nu_3, \quad \nu_{3f}(0) = \nu_3(0) \tag{22}$$

where τ_3 is a time constant.

Step 4) Consider the equation (8). Define the surface error $s_4 = x_5 - \nu_{3f}$. By differentiating it, we obtain

$$\begin{aligned}\bar{L}\dot{s}_4 &= \bar{L}\dot{x}_5 - \bar{L}\dot{\nu}_{3f}\\ &= -Rx_5 - K_e x_4 + \Xi_e(x_e) + u - \bar{L}\dot{\nu}_{3f}.\end{aligned} \tag{23}$$

Then, the actual control law u is chosen as

$$\begin{aligned}u = &-k_4 s_4 + Rx_5 + K_e x_4\\ &- \widehat{\Xi}_e(x_e|\widehat{W}_e) + \hat{\vartheta}(\nu_3 - \nu_{3f})/\tau_3\end{aligned} \tag{24}$$

where k_4 is a positive constant, and $\widehat{W}_e$ and $\hat{\vartheta}$ are the estimated matrices of the matrix $\widehat{W}_e$ and $\vartheta = \bar{L}$, respectively, and are updated by the adaptation laws

$$\dot{\widehat{W}}_{e,i} = \lambda_{3,i}\Theta_{e,i}s_{4,i} - \sigma_3\lambda_{3,i}\widehat{W}_{e,i} \tag{25}$$

$$\dot{\hat{\vartheta}}_i = -\lambda_{4,i}\frac{\nu_{3,i} - \nu_{3f,i}}{\tau_3}s_{4,i} - \sigma_4\lambda_{4,i}\hat{\vartheta}_i \tag{26}$$

where $i = 1,\ldots,n$, λ_3 and λ_4 are tuning gain matrices, and $\sigma_3, \sigma_4 > 0$. Here, $\widehat{W}_{e,i}$, $\hat{\vartheta}_i$, $\lambda_{3,i}$, and $\lambda_{4,i}$ are the ith diagonal element of $\widehat{W}_e$, $\hat{\vartheta}$, λ_3, and λ_4, respectively. $\Theta_{e,i}$, $s_{4,i}$, $\nu_{3,i}$, and $\nu_{3f,i}$ are the ith element of Θ_e, s_4, ν_3, and ν_{3f}, respectively.

C. Stability Analysis

In this subsection, we prove the uniformly ultimately boundedness of the solution of the proposed control system. We first derive analytic expressions of the closed-loop system. Define the boundary layer errors as follows:

$$y_l = \nu_{lf} - \nu_l \tag{27}$$

where $l = 1, 2, 3$.

Using (11), (14), (17), (20), (24) and (27), the derivatives of the error surfaces can be rewritten as follows:

$$\dot{s}_1 = M^{-1}(x_1)K_m(s_2 + y_1) - k_1 s_1 + \widetilde{W}_r^T \Theta_r + \alpha_r, \quad (28)$$

$$\dot{s}_2 = s_3 + y_2 - k_2 s_2, \quad (29)$$

$$\dot{s}_3 = J^{-1}H(s_4 + y_3) - k_3 s_3 + \widetilde{W}_a^T \Theta_a + \alpha_a, \quad (30)$$

$$\bar{L}\dot{s}_4 = -k_4 s_4 + \widetilde{W}_e^T \Theta_e + \alpha_e + \tilde{\vartheta}\frac{y_3}{\tau_3}, \quad (31)$$

where $\tilde{\vartheta} = \vartheta - \hat{\vartheta}$. Differentiating (27), we can obtain

$$\dot{y}_1 = -\frac{y_1}{\tau_1} + P_1(s_1, s_2, y_1, \dot{\widehat{W}}_r, Q_d), \quad (32)$$

$$\dot{y}_2 = -\frac{y_2}{\tau_2} + P_2(s_1, s_2, s_3, y_1, y_2, \dot{\widehat{W}}_r, Q_d), \quad (33)$$

$$\dot{y}_3 = -\frac{y_3}{\tau_3} + P_3(s_1, s_2, s_3, s_4, y_1, y_2, y_3, \dot{\widehat{W}}_r, \dot{\widehat{W}}_a, Q_d), \quad (34)$$

where $Q_d = [q_d\ \dot{q}_d\ \ddot{q}_d]^T$, $P_1(s_1, s_2, y_1, \dot{\widehat{W}}_r, Q_d) = -K_m^{-1}[K_m\dot{x}_1 + F\dot{x}_2 + \frac{\partial G}{\partial x_1}\dot{x}_1 + \frac{\partial C}{\partial x_1}\dot{x}_1 + \frac{\partial C}{\partial x_2}\dot{x}_2 + \frac{\partial M}{\partial x_1}\dot{x}_1\{-k_1 s_1 + \ddot{q}_d - \widehat{\Xi}_r(x_r|\widehat{W}_r) - \Lambda(x_2 - \dot{q}_d)\} + M(x_1)\{-k_1\dot{s}_1 + \dddot{q}_d - \frac{\partial\widehat{\Xi}_r}{\partial x_r}\dot{x}_r - \frac{\partial\widehat{\Xi}_r}{\partial\widehat{W}_r}\dot{\widehat{W}}_r - \Lambda(\dot{x}_2 - \ddot{q}_d)\}]$, $P_2(s_1, s_2, s_3, y_1, y_2, \dot{\widehat{W}}_r, Q_d) = k_2\dot{s}_2 + \frac{\dot{y}_1}{\tau_1}$, and $P_3(s_1, s_2, s_3, s_4, y_1, y_2, y_3, \dot{\widehat{W}}_r, \dot{\widehat{W}}_a, Q_d) = -H^{-1}[K_m(\dot{x}_3 - \dot{x}_1) + B\dot{x}_4 + J\{-k_3\dot{s}_3 - \frac{\partial\widehat{\Xi}_a}{\partial x_a}\dot{x}_a - \frac{\partial\widehat{\Xi}_a}{\partial\widehat{W}_r}\dot{\widehat{W}}_r - \frac{\dot{y}_2}{\tau_2}\}]$ are continuous functions.

Let us consider the following Lyapunov candidate function

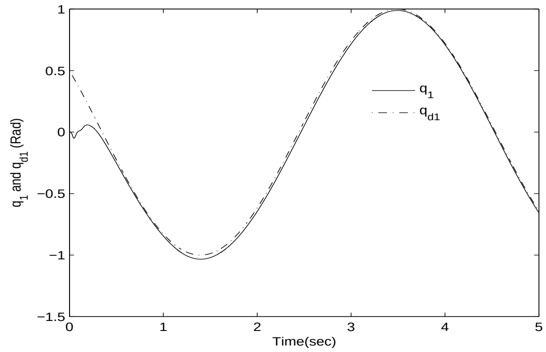
$$V = \frac{1}{2}\Bigg[\sum_{l=1}^{3} s_l^T s_l + s_4^T \bar{L} s_4 + \sum_{l=1}^{3} y_l^T y_l + tr(\widetilde{W}_r^T \lambda_1^{-1}\widetilde{W}_r) + tr(\widetilde{W}_a^T \lambda_2^{-1}\widetilde{W}_a) + tr(\widetilde{W}_e^T \lambda_3^{-1}\widetilde{W}_e) + tr(\tilde{\vartheta}^T \lambda_4^{-1}\tilde{\vartheta})\Bigg], \quad (35)$$

where $\lambda_l = diag[\lambda_{l,i}]$ $(l = 1, \ldots, 4)$, $(i = 1, 2, \cdots, n)$ $\lambda_{l,i}$ are the tuning gains. $tr(\cdot)$ denotes the trace of a matrix.

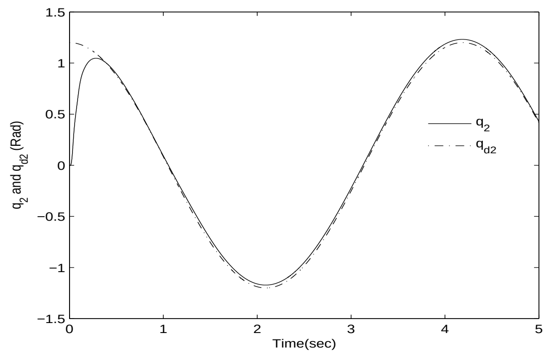
Theorem 1: Suppose that the uncertain EDFJ robot (1), (2), and (3) is controlled by the proposed controller (24). If the proposed control system satisfies *Assumptions* 1-4 and the adaptation laws are chosen as (15), (21), (25), and (26), then for any initial conditions satisfying $V(0) \leq \mu$, there exist k_l, σ_l, and λ_l $(l = 1, 2, 3, 4)$ such that the errors of states and adjustable weights of the closed-loop system are uniformly ultimately bounded and may be kept arbitrarily small.

Proof: See the Appendix I. ■

Remark 1: In the adaptation laws (15), (21), and (25), the partial derivative terms $\Theta_{r,i}$, $\Theta_{a,i}$, and $\Theta_{e,i}$ for tuning all weights of the SRWNNs can be evaluated by the chain rule, as illustrated in [14].



(a)



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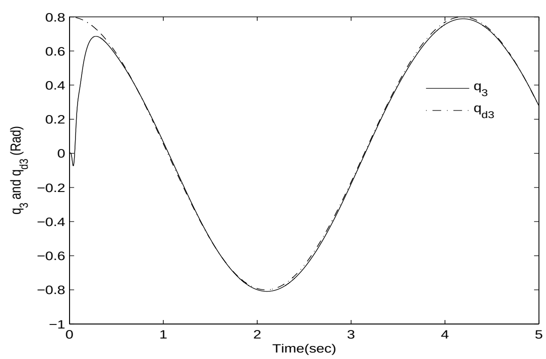
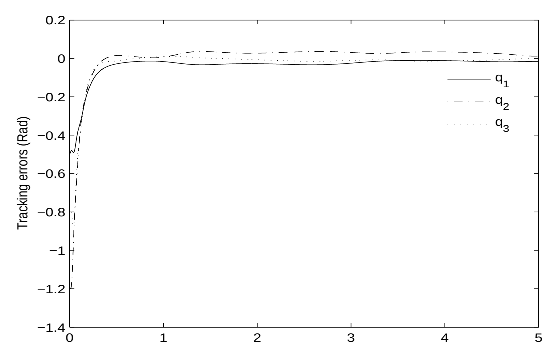


(c)



(d)

Fig. 1. Tracking results and errors (a) Joint 1 (b) Joint 2 (c) Joint 3 (d) Tracking errors.

TABLE I
THE NOMINAL PARAMETERS FOR THE ROBOT DYNAMICS.

	Mass (kg)	Link (m)	Moment of Inertia (kgm^2)
Joint 1	1.0	0.5	43.33$\times 10^{-3}$
Joint 2	0.7	0.4	25.08$\times 10^{-3}$
Joint 3	1.4	0.3	32.67$\times 10^{-3}$

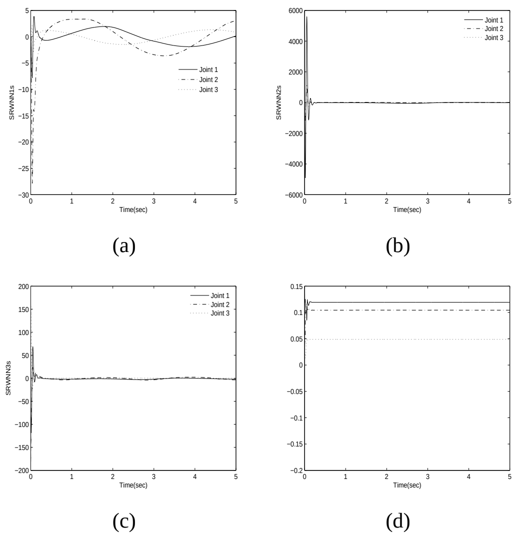


Fig. 2. Outputs of SRWNNs and estimates of diagonal elements of ϑ (a) SRWNN1 (b) SRWNN2 (c) SRWNN3 (d) $\hat{\vartheta}$.

IV. Simulation Results

In this section, to illustrate the validity of the suggested adaptive control system, the three-link EDFJ manipulator is considered. The nominal robot dynamics used in [15] is used. The nominal robot parameters of the three-link EDFJ manipulator are defined in Table I. In this simulation, the link masses m_is in the robot dynamics, and the parameters K_m, H, R, and K_e in the actuator dynamics are assumed to be uncertain. It is assumed that the masses in the robot dynamics have 50%, 100%, and 50% uncertainties, namely, the actual mass values $\bar{m}_i$ with uncertainties are $\bar{m}_1 = 1.5$, $\bar{m}_2 = 1.4$, and $\bar{m}_3 = 2.1$. The nominal and actual EDFJ parameters are given as J = diag$[0.03\ 0.03\ 0.03]$, B = diag$[5\ 5\ 5]$, K_m = diag$[100\ 100\ 100]$, H = diag$[10\ 10\ 10]$, R = diag$[1.2\ 1.2\ 1.2]$, K_e = diag$[15.6\ 15.6\ 15.6]$, $\bar{K}_m$ = diag$[106.2\ 105.6\ 103.2]$, $\bar{H}$ = diag$[10.5\ 11.2\ 11.1]$, $\bar{L}$ = diag$[0.048\ 0.048\ 0.048]$, $\bar{R}$ = diag$[2.3\ 1.5\ 2.5]$, and $\bar{K}_e$ = diag$[15.2\ 15.9\ 16.1]$. In addition, the time-varying external disturbances T_r, T_a, and T_e given by

$$T_r = [0.2\cos(t) \quad 0.2\sin(t) \quad 0.1\cos(t)]^T,$$
$$T_a = [0.1\cos(t) \quad 0.2\sin(2t) \quad 0.2\cos(t)]^T,$$

and

$$T_e = [0.1\cos(2t) \quad 0.2\sin(1.5t) \quad 0.2\cos(t)]^T,$$

are assumed to influence the actual EDFJ robot model. In this simulation, the initial positions of the three-link FJ manipulator are set to $q_1(0) = q_2(0) = q_3(0) = 0$ and the controller parameters for the proposed control system are chosen as $k_l = 60$, $\Lambda = diag[8 \quad 10 \quad 15]$, $\lambda_j = diag[0.00001 \quad 0.00002 \quad 0.0001]$, $\lambda_4 = diag[0.000002 \quad 0.000002 \quad 0.000002]$, $\sigma_l = 0.01$, and $\tau_j = 0.001$ $(l = 1, \ldots, 4)$ $(j = 1, 2, 3)$. We employ the SRWNN1 system $\widehat{\Xi}_r(\cdot)$, the SRWNN2 system $\widehat{\Xi}_a(\cdot)$, and the SRWNN3 system $\widehat{\Xi}_e(\cdot)$. Here, note that only one product node is used for each SRWNN. The tracking results and errors of the the proposed control system as shown in Fig. 1 indicate that the suggested method can overcome unknown model uncertainties resulting from the robot dynamics, joint flexibility, and the motor dynamics, and time-varying external disturbances. Figure 2 displays the outputs of the SRWNNs and estimates of diagonal elements of ϑ. Note that the uncertainty terms ($\Xi_r(\cdot)$, $\Xi_a(\cdot)$, $\Xi_e(\cdot)$, ϑ) are approximated by SRWNNs and the adaptive technique, effectively. Besides, we can see that all signals in the closed-loop system are bounded.

V. Conclusion

In this paper, a simple adaptive control system for the EDFJ robot with model uncertainties has been developed. First, the dynamics of the EDFJ robots has been introduced. Second, the simple control law using the DSC technique and SRWNNs has been designed for the tracking control of EDFJ robots with model uncertainties and external disturbances. Third, from Lyapunov stability analysis, it is proved that all signals in the closed-loop system are uniformly ultimately bounded. Finally, from the simulation results for three-link EDFJ manipulator, it was shown that the proposed control system has the good tracking performance and the robustness against model uncertainties and external disturbances.

Appendix I
The proof of Theorem 1.

Differentiating the Lyapunov candidate function (35) and substituting (15), (21), (25), and (26), (28)-

(34), we can obtain

$$\begin{aligned}\dot{V} &= s_1^T(M^{-1}K_m(s_2+y_1) - k_1 s_1 + \alpha_r) \\ &+ s_2^T(s_3+y_2-k_2 s_2) \\ &+ s_3^T(J^{-1}H(s_4+y_3) - k_3 s_3 + \alpha_a) \\ &+ s_4^T(-k_4 s_4 + \alpha_e) + \sigma_1 tr(\widetilde{W}_r^T \widehat{W}_r) \\ &+ \sigma_2 tr(\widetilde{W}_a^T \widehat{W}_a) + \sigma_3 tr(\widetilde{W}_e^T \widehat{W}_e) + \sigma_4 tr(\tilde{\vartheta}_r^T \tilde{\vartheta}_r) \\ &+ \sum_{l=1}^{3} y_l^T \left(-\frac{y_l}{\tau_l} + P_l\right). \end{aligned} \quad (36)$$

From the boundedness of Q_d and the existence of μ, there exists a positive constant R_l such that $\|P_l\| \le R_l$. Therefore, using Property 2, *Assumption* 3, (12), and the fact $2z_1z_2 \le z_1^2 + z_2^2$ yields

$$\begin{aligned}\dot{V} &\le \left(\frac{1}{2}+M_m\right)\|s_1\|^2 + \left(1+\frac{M_m}{2}\right)\|s_2\|^2 \\ &+ (J_m+1)\|s_3\|^2 + \left(\frac{1+J_m}{2}\right)\|s_4\|^2 \\ &+ \left(1+\frac{M_m}{2}\right)\|y_1\|^2 + \frac{3}{2}\|y_2\|^2 \\ &+ \left(1+\frac{J_m}{2}\right)\|y_3\|^2 + \frac{1}{2}\rho_r^2 + \frac{1}{2}\rho_a^2 + \frac{1}{2}\rho_e^2 \\ &- \sum_{l=1}^{4} k_l\|s_l\|^2 + \sum_{l=1}^{3}\left(-\frac{1}{\tau_l}\|y_l\|^2 + \frac{1}{4}R_l^2\right) \\ &- \frac{1}{2}\sigma_1\|\widetilde{W}_r\|_F^2 - \frac{1}{2}\sigma_2\|\widetilde{W}_a\|_F^2 - \frac{1}{2}\sigma_3\|\widetilde{W}_e\|_F^2 \\ &- \frac{1}{2}\sigma_4\|\tilde{\vartheta}\|_F^2 + \frac{1}{2}\sigma_1 W_{r,M}^2 + \frac{1}{2}\sigma_2 W_{a,M}^2 \\ &+ \frac{1}{2}\sigma_3 W_{e,M}^2 + \frac{1}{2}\sigma_4\|\vartheta\|_F^2 \end{aligned}$$

where J_m is a maximum eigenvalue of $J^{-1}H$. Here, choosing $k_1 = (1/2) + M_m + k_1^*$, $k_2 = 1 + (M_m/2) + k_2^*$, $k_3 = J_m + 1 + k_3^*$, $k_4 = (1+J_m)/2 + k_4^*$, $1/\tau_1 = 1 + (M_m/2) + \gamma_1$, $1/\tau_2 = (3/2) + \gamma_2$, and $1/\tau_3 = 1 + (J_m/2) + \gamma_3$,

$$\begin{aligned}\dot{V} &\le -\sum_{l=1}^{4} k_l^*\|s_l\|^2 - \frac{1}{2}\sigma_1\|\widetilde{W}_r\|_F^2 - \frac{1}{2}\sigma_2\|\widetilde{W}_a\|_F^2 \\ &- \frac{1}{2}\sigma_3\|\widetilde{W}_e\|_F^2 - \frac{1}{2}\sigma_4\|\tilde{\vartheta}\|_F^2 - \sum_{l=1}^{3}\gamma_l\|y_l\|^2 + O \\ &\le -2\zeta V + O. \end{aligned} \quad (37)$$

where $k_l^* > 0$, $\gamma_j > 0$, $O = \frac{1}{2}\sigma_1 W_{r,M}^2 + \frac{1}{2}\sigma_2 W_{a,M}^2 + \frac{1}{2}\sigma_3 W_{e,M}^2 + \frac{1}{2}\sigma_4\|\vartheta\|_F^2 + \frac{1}{2}\rho_r^2 + \frac{1}{2}\rho_a^2 + \frac{1}{2}\rho_e^2 + \frac{1}{4}\sum_{l=1}^{3} R_l^2$, and $0 < \zeta < min[k_1^*, k_2^*, k_3^*, k_4^*/L_M, \gamma_1, \gamma_2, \gamma_3, \frac{\sigma_1\lambda_{1,m}}{2}, \frac{\sigma_2\lambda_{2,m}}{2}, \frac{\sigma_3\lambda_{3,m}}{2}, \frac{\sigma_4\lambda_{4,m}}{2}]$. Here $\lambda_{1,m}$, $\lambda_{2,m}$, $\lambda_{3,m}$, and $\lambda_{4,m}$ are the minimum eigenvalues of λ_1, λ_2, λ_3, and λ_4, respectively. Equation (37) implies that $\dot{V} < 0$ on $V = \mu$ when $V > (O/2\zeta)$. Accordingly, all signals in the controlled closed-loop system are uniformly ultimately bounded. Besides, the errors can be kept arbitrarily small by adjusting K_l^*, γ_j, σ_l, λ_l $(l = 1, \cdots, 4)$, $(j = 1, 2, 3)$. That is, the tracking error s_1 can be made arbitrarily small. This completes the proof of the theorem. ■

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